Week 5: Derivatives: Theorems and Critical Points

1. MULTI Single

Which of the following functions is guaranteed to have a critical point with $f'(c) = 0$ by Rolle's Theorem?

(a) $f(t) = \log(t)$
(b) $f(x) = x^3$
(c) $f(x) = a^x$
(d) $f(t) = 2t^3 - t^2 - t$

2. MULTI Single

Given $f(x) = 2x^3 - 9x^2 - 24x + a$ and knowing that the equation $f(x) = 0$ has 3 distinct real solutions $x_1 < x_2 < x_3 \in \mathbb{R}$ which of the following is always true?

(a) $x_1 \in (-\infty, -1), x_2 \in (-4, 1), x_3 \in (4, \infty)$
(b) $x_1 \in (-\infty, -1), x_2 \in (-1, 4), x_3 \in (4, \infty)$
(c) $x_1 \in (-\infty, -2), x_2 \in (-3, 2), x_3 \in (3, \infty)$
(d) $x_1 \in (-\infty, -2), x_2 \in (-2, 3), x_3 \in (3, \infty)$

3. MULTI Single

Which of the following is not a requirement for the Mean Value Theorem to hold?

(a) f must be differentiable on (a, b)
(b) The derivative of f must be continuous on (a, b)
(c) f must be continuous on $[a, b]$
(d) f must be a function from $[a, b]$ to $\mathbb{R}$

4. MULTI Single

Let $f(x) = \dfrac{x}{1+x}$. What is the value of c over the interval $(0, 3)$ such that the Mean Value Theorem is satisfied?

(a) $c = \dfrac{1}{4}$
(b) The MVT does not apply in this case
(c) $c = 1$
(d) $c = -3$

5. MULTI Single

Which side lengths of a rectangle with a perimeter of $4L$ maximize its area?

(a) $(\frac{1}{4}L, \frac{7}{4}L)$
(b) (L, L)
(c) $(\frac{3}{2}L, \frac{1}{2}L)$
(d) $(0.01L, 1.99L)$

6. MULTI Single

An algorithm can calculate the universe partition function (a function one can give to a statistical physicist to find out the secrets of our universe) with a precision

level N in time $T(N) = a^2/2 - (e^N a \sin(2N))/(\cos(2N) + 1)$. The parameter a corresponds to a specific choice of hyperparameters in the algorithm. For a given $N \in Dom(T)$, which a does one need to choose to use the fastest version of the algorithm?

(a) $a = \dfrac{e^N \sin(x)}{2}$
(b) $a = \dfrac{e^N \ln(N)}{2}$
(c) $a = e^N \tan(N)$
(d) $a = \sin(N)\cos(N)e^N$

7. MULTI Single

What are the maxima and minima of the function $f(x) = \dfrac{1}{3}x^3 - \dfrac{7}{2}x^2 + 10x + 3$?

(a) At $x = 3$ there is a minimum, at $x = 1$ a maximum.
(b) There are no maxima or minima.
(c) At $x = 2$ there is a minimum, at $x = 5$ a maximum.
(d) At $x = 2$ there is a maximum, at $x = 5$ a minimum.

8. MULTI Single

For which interval is $f(x) = \dfrac{x^2}{\pi^2 - x^2}$ positive?

(a) $x \in (-\infty, 0)$
(b) $x \in (-\pi, 0) \cup (\pi, \infty)$
(c) $x \in (-\pi, \pi)$
(d) $x \in (-\infty, -\pi) \cup (\pi, \infty)$

9. MULTI Single

For which values of x does $f(x) = -\ln(x) + \sqrt{x}$ have maxima or minima?

(a) No maxima or minima
(b) $x = -2$ is a maximum and $x = 2$ is a minimum
(c) $x = 4$ is a minimum
(d) $x = 2$ is a maximum

10. MULTI Single

For which value of x does $f(x) = 2\mathrm{e}^{-4/x}$ have a point of inflection?

(a) $x = 2$
(b) There is no point of inflection
(c) $x = -4$
(d) $x = 32$

Total of marks: 10