Week 6: Taylor Series and Indefinite Integrals

1. MULTI Single

   What are the first three non-zero terms in the Taylor series of $f(x) = e^{-x^2}\cos(x)$ centred at $b = 0$?

   (a) $1,\ -\frac{1}{2}x^2,\ \frac{1}{24}x^4$
   (b) $1,\ -x^2,\ \frac{1}{2}x^4$
   (c) $1,\ \frac{x^4}{2},\ \frac{x^8}{48}$
   (d) $1,\ -\frac{3}{2}x^2,\ \frac{25}{24}x^4$

2. MULTI Single

   The hyperbolic sine is defined by $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$. What is the n-th term in the Taylor expansion of $\sinh(x)$ around $b = 0$?

   (a) $\frac{(-1)^n}{(2n+1)!}x^{2n+1}$
   (b) $\frac{x^{2n+1}}{(2n+1)!}$
   (c) $\frac{(-1)^n}{(2n)!}x^{2n}$
   (d) $\frac{x^{2n}}{(2n)!}$

3. MULTI Single

   Let F_N be the N-th order Taylor polynomial of e^x centered at $b = 0$. What is the smallest value of N for which $F_N(1)$ approximates e to two decimal places?

   (a) $N = 5$
   (b) $N = 3$
   (c) $N = 4$
   (d) $N = 6$

4. MULTI Single

   Use Newton's method on $x^3+x+3 = 0$ to compute x_2 when the initial approximation is $x_1 = -1$.

   (a) 1
   (b) 3/4
   (c) −5/4
   (d) −3/2

5. MULTI Single

   Evaluate $\int \frac{\cos(\pi/x)}{x^2}\,\mathrm{d}x$. (*Hint*: substitute $\frac{\pi}{x}$.)

(a) $\frac{1}{\pi}\sin\frac{1}{x}+C$
(b) $-\frac{1}{\pi}\sin\frac{\pi}{x}+C$
(c) $-\frac{1}{\pi}\sin \pi x+C$
(d) $\sin\frac{\pi}{x}+C$

6. MULTI Single

Compute $\int \frac{1}{\sqrt{9-x^2}}\,\mathrm{d}x$. *Hint*: How about a substitution involving the sine?

(a) $2\sqrt{9-x^2}+C$
(b) $\sin\left(\frac{x}{3}\right)+C$
(c) $\sin^{-1}\left(\frac{x}{3}\right)+C$
(d) $\cos^{-1}\left(\frac{x}{3}\right)+C$

7. MULTI Single

Evaluate $\int \sin(x)\ln(\cos x)\,\mathrm{d}x$ (*Hint:* use integration by parts)

(a) $\cos x(1+\ln\cos x)+C$
(b) $\cos x(1+\ln\cos x)$
(c) $\cos x(1-\ln\cos x)$
(d) $\cos x(1-\ln\cos x)+C$

8. MULTI Single

Compute $\int x^n e^x \mathrm{d}x$ for $n\in\mathbb{N}$.

(a) $\left(\sum_{k=0}^{n}\frac{n!\,x^{n-k}}{(n-k)!}\right)\mathrm{e}^x$
(b) $\left(\sum_{k=0}^{n}\frac{x^{n-k}}{(n-k)!}\right)\mathrm{e}^x$
(c) $\left(\sum_{k=0}^{n}(-1)^k\frac{n!\,x^{n-k}}{(n-k)!}\right)\mathrm{e}^x$
(d) $\left(\sum_{k=0}^{n}k!\,x^{n-k}\right)\mathrm{e}^x$

9. MULTI Single

Given $I_n=\int_0^1(a-bx^3)^n$, find a relationship between I_n and I_{n-1}.

(a) $I_n=\frac{n}{n+1}I_{n-1}^3$
(b) $I_n=\left(\frac{n}{n+1}\right)^3 I_{n-1}$

(c) $I_n = \frac{3n}{3n+1} I_{n-1}$

(d) $I_n = \frac{3n}{3n-1} I_{n-1}$

10. MULTI Single

Evaluate $\int \sqrt{1-x^2}\,\mathrm{d}x$. *Hint:* Use a trigonometric substitution.

(a) $\frac{x\sqrt{1-x^2}}{2} + \frac{\arcsin(x)}{2} + C$

(b) $\sqrt{x} + \arctan(x) + C$

(c) $\frac{\tan(2x)}{2} + \frac{xe^x}{2} + C$

(d) $\frac{1}{\sqrt{1-x^2}} + \cos(x) + C$

Total of marks: 10