Week 7: Definite Integrals, the FTC, and Applications of Integration

1. MULTI Single

   Calculate $\displaystyle\int_2^1 \frac{2y^3 - 6y^2}{y^2}\,\mathrm{d}y$.

   (a) 9
   (b) -3
   (c) -9
   (d) 3

2. MULTI Single

   Find $\displaystyle\int_1^{\mathrm{e}} \frac{\ln x}{x}\,\mathrm{d}x$. (*Hint:* use a substitution.)

   (a) 1
   (b) 0.5
   (c) 0.75
   (d) 1.5

3. MULTI Single

   Calculate $\displaystyle\int_0^{\pi/2} x\sin(x)\cos(x)\,\mathrm{d}x$. (*Hint:* simplify $\sin(x)\cos(x)$ using a trigonometric identity, and then use integration by parts.)

   (a) 0
   (b) $\pi/8$
   (c) $\pi/4$
   (d) $3\pi/8$

4. MULTI Single

   Calculate $\displaystyle\int_{-1}^{1} f(x)\,\mathrm{d}x$ where

   $$f(x) = x\left(\frac{e^x - e^{-x}}{2}\right)\tan(x)$$

   (a) e
   (b) 2
   (c) $-\pi$
   (d) 0

5. MULTI Single

   Calculate $\displaystyle\int_0^{1/2} \frac{2x^2+2}{x^2-1}\,\mathrm{d}x$

   (a) -1
   (b) $1 - 2\ln(3)$

(c) $2\ln(3)$
(d) $2\ln(2) - 1$

6. MULTI Single

Evaluate $\displaystyle\int_0^1 \frac{3x^2 + 12x + 11}{(x+1)(x+2)(x+3)}\,\mathrm{d}x$

(a) $4\ln(2) - 2$
(b) $-\ln(5)$
(c) $2\ln(2)$
(d) $\ln(3)$

7. MULTI Single

Find the area A under the curve of $f(x) = \sqrt{x}$ from $x = 0$ to $x = 4$.

(a) $A = \dfrac{1}{4}$
(b) $A = 2$
(c) $A = 8$
(d) $A = \dfrac{16}{3}$

8. MULTI Single

Calculate the area between $\sin(x)$ and $\cos(x)$ on the interval $[0, 2\pi]$. *Hint*: $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, $\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, $\sin\left(\frac{5\pi}{4}\right) = \frac{-1}{\sqrt{2}}$, $\cos\left(\frac{5\pi}{4}\right) = \frac{-1}{\sqrt{2}}$.

(a) $\sqrt{2}$
(b) $2\sqrt{2}$
(c) $4\sqrt{2}$
(d) 0

9. MULTI Single

Find the area between the curves $x = 1 - y^2$ and $y = -x - 1$.

(a) 1
(b) 2
(c) 4.5
(d) 3.5

10. MULTI Single

Which of the following integrals computes the volume V of a cone of height h and base radius R?

(a) $V = \displaystyle\int_0^h A(x)\,\mathrm{d}x$ with $A(x) = \pi\dfrac{R^2}{h^2}x^2$.
(b) $V = \displaystyle\int_0^h A(x)\,\mathrm{d}x$ with $A(x) = \dfrac{1}{3}\pi R^2 h$.
(c) $V = \displaystyle\int_0^h A(x)\,\mathrm{d}x$ with $A(x) = \pi\dfrac{h^2}{R^2}x^2$.

(d) $V = \int_0^R A(x)\,\mathrm{d}x$ with $A(x) = \pi x^2$.

Total of marks: 10