Week 9: Finite Difference Schemes and Total/Directional/Partial Derivatives

1. MULTI Single

   Which of the following statements about explicit and implicit finite difference schemes is false?

   (a) Implicit scheme are usually unconditionally stable.
   (b) Implicit schemes are more computationally expensive.
   (c) Explicit scheme usually have stability conditions.
   (d) Explicit schemes are more computationally expensive.

2. MULTI Single

   Discretize the ODE
   $$\frac{\mathrm{d}y}{\mathrm{d}t} = -y + t$$
   using the Backward Euler method and write out the iteration scheme.

   (a) $y_{n+1} = \dfrac{y_n + (\Delta t)t_{n+1}}{\Delta t}$
   (b) $y_{n+1} = \dfrac{y_n + (\Delta t)t_{n+1}}{1 + \Delta t}$
   (c) $y_{n+1} = y_n - (\Delta t)(-y_n - t_n)$
   (d) $y_{n+1} = y_n + (\Delta t)(-y_n + t_n)$

3. MULTI Single

   Discretize the ODE
   $$\frac{\mathrm{d}y}{\mathrm{d}t} = -y + t$$
   using the Forward Euler method and write out the iteration scheme.

   (a) $y_{n+1} = \dfrac{y_n + (\Delta t)t_{n+1}}{\Delta t}$
   (b) $y_{n+1} = \dfrac{y_n + (\Delta t)t_{n+1}}{1 + \Delta t}$
   (c) $y_{n+1} = y_n + (\Delta t)(-y_n + t_n)$
   (d) $y_{n+1} = y_n - (\Delta t)(-y_n - t_n)$

4. MULTI Single

   The Crank-Nicolson method for discretizing first order ODEs
   $$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y, t)$$
   uses the forward difference quotient for the derivative and evaluates the right-hand side at the average of the current and the next time step, i.e., it discretizes the ODE as
   $$\frac{y_{n+1} - y_n}{\Delta t} = \frac{1}{2}\Big(f(y_n, t_n) + f(y_{n+1}, t_{n+1})\Big).$$
   Use the Crank-Nicolson method to discretize the ODE
   $$\frac{\mathrm{d}y}{\mathrm{d}t} = -y + t$$
   and write out the iteration scheme.

(a) $y_{n+1} = \dfrac{y_n - \frac{(\Delta t)}{2}(t_n + t_{n+1})}{1 + \frac{(\Delta t)}{2}}$

(b) $y_{n+1} = \dfrac{y_n + \frac{(\Delta t)}{2}(t_n + t_{n+1})}{1 - \frac{(\Delta t)}{2}}$

(c) $y_{n+1} = \dfrac{y_n + \frac{(\Delta t)}{2}(t_n + t_{n+1})}{1 + \frac{(\Delta t)}{2}}$

(d) $y_{n+1} = \dfrac{y_n + (\Delta t)(t_n + t_{n+1})}{1 + \Delta t}$

5. MULTI Single

Compute the total derivative of $f(x, y, z) = x^2 + xy + yz$.

(a) The total derivative is $\begin{pmatrix} 2x + y & x + z \\ y & 2x + y \end{pmatrix}$.
(b) The total derivative is $(2x + y, x + z, y)$.
(c) The total derivative is $(2x, x, y)$.
(d) The total derivative is $\begin{pmatrix} x & y \\ y & y + z \end{pmatrix}$.

6. MULTI Single

Compute the total derivative of $f(x, y) = \begin{pmatrix} x^2 + y \\ 2xy \end{pmatrix}$.

(a) The total derivative is $\begin{pmatrix} 1 & 2x \\ 2x & 2y \end{pmatrix}$.
(b) The total derivative is $\begin{pmatrix} 2x & 1 \\ 2y & 2x \end{pmatrix}$.
(c) The total derivative is $(2x, 2y)$.
(d) The total derivative is $\begin{pmatrix} 2x \\ 2y \end{pmatrix}$.

7. MULTI Single

Let $f(x, y) = \dfrac{x - y}{x + y}$. What is $\dfrac{\partial f}{\partial x}$?

(a) $\dfrac{2}{(x + y)^2}$
(b) $\dfrac{2x}{(x + y)}$
(c) $\dfrac{2y}{(x + y)}$
(d) $\dfrac{2y}{(x + y)^2}$

8. MULTI Single

Let $f(x, y, z) = \ln(x + 2y + 3z)$. What is $\dfrac{\partial f}{\partial z}$?

(a) 3

(b) $\dfrac{1}{x+2y+3z}$
(c) $\dfrac{3}{x+2y+3z}$
(d) $3\ln(x+2y+3z)$

9. MULTI Single

The total resistance R produced by three conductors with resistances R_1, R_2, R_3 connected in a parallel circuit is given by the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

What is $\dfrac{\partial R}{\partial R_1}$?

(a) $-R_2^2$
(b) $-\dfrac{1}{R_2^2}$
(c) $\dfrac{R^2}{R_1^2}$
(d) $\dfrac{R}{R_1}$

10. MULTI Single

The gas law for a fixed mass m of an ideal gas at absolute temperate T, pressure P, and volume V is $PV = mRT$, where R is a constant. What is the value of $T \cdot \dfrac{\partial P}{\partial T} \cdot \dfrac{\partial V}{\partial T}$?

(a) mR
(b) $(mR)^2$
(c) $\dfrac{PV}{(mR)^2}$
(d) PV/T

Total of marks: 10