Week 10: Total Directional Partial Derivatives

1. MULTI Single

   Let $z(x,y) = x^2y^3$ and $x(s,t) = s\cos(t)$ and $y(s,t) = s\sin(t)$. Compute the partial derivatives $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

   (a) $\frac{\partial z}{\partial s} = \cos(t), \frac{\partial z}{\partial t} = -s\sin(t)$
   (b) $\frac{\partial z}{\partial s} = 3x^2y^2\sin(t), \frac{\partial z}{\partial t} = -3sx^2y^2\cos(t)$
   (c) $\frac{\partial z}{\partial s} = \sin(t), \frac{\partial z}{\partial t} = s\cos(t)$
   (d) $\frac{\partial z}{\partial s} = 2xy^3\cos(t) + 3x^2y^2\sin(t), \frac{\partial z}{\partial t} = -2sxy^3\sin(t) + 3sx^2y^2\cos(t)$

2. MULTI Single

   Let $z(x,y) = e^x\cos(y)$ and $x(s,t) = st$ and $y(s,t) = \sqrt{s^2+t^2}$. Compute the partial derivatives $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

   (a) $\frac{\partial z}{\partial s} = t, \frac{\partial z}{\partial t} = s$
   (b) $\frac{\partial z}{\partial s} = te^x\cos(y) - e^x\sin(y)\cdot\frac{s}{\sqrt{s^2+t^2}}, \frac{\partial z}{\partial t} = se^x\cos(y) - e^x\sin(y)\cdot\frac{t}{\sqrt{s^2+t^2}}$
   (c) The partials don't exist
   (d) $\frac{\partial z}{\partial s} = e^{st}\cos(\sqrt{s^2+t^2}), \frac{\partial z}{\partial t} = -e^{st}\cos(\sqrt{s^2+t^2})$

3. MULTI Single

   Suppose that over a region of space the electric potential V is given by $V(x,y,z) = 5x^2 - 3xy + xyz$. What is the rate of change (i.e., the gradient) of the potential at $(3,4,5)$?

   (a) $(10,-3,1)$
   (b) $(38,6,12)$
   (c) $(45,-60,60)$
   (d) $(30,15,12)$

4. MULTI Single

   What is the gradient of $f(x,y) = \frac{y^2}{x}$ at $(2,4)$?

   (a) $(2,4)$
   (b) $(2,2)$
   (c) $(-4,4)$
   (d) $(1,-1)$

5. MULTI Single

   What is the gradient of $f(x,y,z) = xe^y\cos(z) - z - 1$ at $(1,0,0)$?

   (a) $(-1,-1,1)$

(b) $(1, 1, -1)$
(c) $(1, 0, 1)$
(d) $(-1, 0, 1)$

6. MULTI Single

Let $f(x, y) = 1 + 2x\sqrt{y}$. Find the directional derivative at $(3, 4)$ in the direction $\vec{v} = (4, -3)$.

(a) $3/2$
(b) 4
(c) $23/10$
(d) $23/2$

7. MULTI Single

Let $f(x, y, z) = xe^y + ye^z + ze^x$. Find the directional derivative at $(0, 0, 0)$ in the direction $\vec{v} = (5, 1, 2)$.

(a) $2/15$
(b) 4
(c) $4/\sqrt{30}$
(d) $8/\sqrt{30}$

8. MULTI Single

Consider $f(x, y) = \sin(xy)$ at the point $(1, 0)$. In which direction does f have the maximum rate of change?

(a) $(1, 0)$
(b) $(-1, 0)$
(c) $(0, 1)$
(d) $(0, -1)$

9. MULTI Single

Let $f(x, y) = e^{x^2+y}$. Find the second order Taylor polynomial that approximates f at $(0, 0)$.

(a) $1 + x + y + \frac{x^2}{2} + \frac{xy}{2} + \cdots$
(b) $x + y + \frac{x^2}{2} + \frac{y^2}{2} + \frac{xy}{2} + \cdots$
(c) $1 + y + x^2 + \frac{y^2}{2} + \cdots$
(d) $1 + y + x^2 + \frac{y^2}{2} + \frac{xy}{2} + \cdots$

10. MULTI Single

Compute the derivative $\frac{d}{dx}\int_0^1 (2t + x^3)^2 dt$.

(a) $6x^2 + 6x^5$
(b) $12x^2 + 6x^5$

(c) $12x^2$
(d) $x^2 + x^5$

Total of marks: 10