Week 12: Lagrange Multipliers, Least Squares, Gradient Descent

1. MULTI Single

   Consider a function $G : \mathbb{R}^n \to \mathbb{R}$ that is once continuously differentiable and its level set $\{x \in \mathbb{R}^n : G(x) = 0\}$. Which of the following is true?

   (a) The gradient of G is parallel to S at any point in S.
   (b) The level set might not be well-defined.
   (c) The gradient of G is orthogonal to S at any point in S.
   (d) The gradient of G is orthogonal to S at any point in S.

2. MULTI Single

   Use the method of Lagrange multipliers to find the extreme values of the function $f(x, y) = x + y$ subject to the constraint $x^2 + 4y^2 = 4$.

   (a) There are no maximum or minimum values.
   (b) The maximum value is $\sqrt{5}$, the minimum value is $-\sqrt{5}$.
   (c) There is only one maximum value at $\sqrt{5}$ but no minimum value.
   (d) The maximum value is $\sqrt{2}$, the minimum value is $-\sqrt{2}$.

3. MULTI Single

   Use the method of Lagrange multipliers to find the extreme values of the function $f(x, y) = xy$ subject to the constraint $x^2 + y^2 = 2$.

   (a) There are no maximum or minimum values.
   (b) The maximum value is 1, the minimum value is -1.
   (c) The maximum value is $\sqrt{2}$, the minimum value is $-\sqrt{2}$.
   (d) The maximum value is 2, the minimum value is -2.

4. MULTI Single

   Use the method of Lagrange multipliers to find the extreme values of the function $f(x, y) = x^2 + y^2$ subject to the constraint $xy = 1$.

   (a) The function has no minimum value and a maximum value of 4
   (b) The function has no minimum or maximum values.
   (c) The function has a minimum value of 2 and no maximum value.
   (d) The function has a minimum value of 2 and a maximum value of 4.

5. MULTI Single

   Use the method of Lagrange multipliers to find the extreme values of the function $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $x^4 + y^4 + z^4 = 1$.

   (a) The function has a maximum value of $\sqrt{2}$ and a minimum value of 1.
   (b) The function has a maximum value of $\sqrt{3}$ and a minimum value of 1.
   (c) The function has a maximum value of $\sqrt{3}$ and no minimum value.
   (d) The function has no maximum value and a minimum value of 1

6. MULTI Single

   Let us reconsider the following exercise from last week: Find three positive numbers whose sum is 100 and whose product is maximal. What is the product? Solve this question using the method of Lagrange multipliers.

(a) $(9.7 \times 10^5)/27$
(b) (3.6×10^4)
(c) $10^6/27$
(d) $(2.2 \times 10^6)/91$

7. MULTI Single

How do we find the solution to the least-square problem, i.e., to minimize $\|Ax - b\|$, where A is an $m \times n$ matrix with $m > n$, $b \in \mathbb{R}^m$?

(a) $x = A^{-1}b$, where A^{-1} is the inverse of A.
(b) Solve the system of linear equations $Ax = b$ directly.
(c) Solve the system of linear equations $AA^T x = A^T b$.
(d) Solve the system of linear equations $A^T Ax = A^T b$.

8. MULTI Single

Let $f : \mathbb{R}^n \to \mathbb{R}$ and choose an initial point $a_0 \in \mathbb{R}^n$ and a step size $\eta > 0$. Which of the following iterations is the method of gradient descent?

(a) $a_{n+1} = a_n + \eta(\nabla f)(a_n)$
(b) $a_{n+1} = a_n - \eta f(a_n)$
(c) $a_{n+1} = a_n - \eta(\nabla f)(a_n)$
(d) $a_{n+1} = -\eta f(a_n)$

9. MULTI Single

Which of the following is a problem of the method of gradient descent?

(a) It cannot find local minima, only global ones.
(b) It sometimes finds maxima instead of minima.
(c) It might overshoot a minimum.
(d) It is generally regarded as a very slow method.

10. MULTI Single

Which of the following is an advantage of stochastic gradient descent?

(a) It is computationally cheap.
(b) It uses a very large learning rate.
(c) It might jump around an actual global minimum.
(d) It often needs more steps than regular gradient descent to find a minimum.

Total of marks: 10